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Modeling viral dynamics of Lassa fever transmission: A Caputo-fractional order study

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Abstract Lassa fever, proclaimed as endemic in West Africa, is emanated by Lassa virus that is carried by infected Mastomys rats. In this article, we propose a five compartmental deterministic ODE model in studying of Lassa fever contagion dynamics considering the role of both the vector (Mastomys rats) population and human population. The epidemic system executes one disease-free steady state and one endemic steady state. Basic reproduction number of the epidemic system is computed. Furthermore, the epidemic model is upgraded to its fractional-order counterpart in Caputo sense. Local dynamics of the epidemic system is studied around both the steady states. Numerical simulations are performed with reference to the real epidemic cases of Lassa fever (LF) in Nigeria to investigate its possible prevention and control. Analytical results are validated epidemiologically.

1 Introduction:

Lassa Fever (LF), acknowledged precisely as Lassa Hemorrhagic Fever (LHF), declared as an endemic contagion in some parts of West African countries, is spread by Lassa virus (LASV) or Lassa Hemorrhagic Fever virus (LASHFV), belonging to the Arenaviridae family. The Lassa fever is a zoonotic infection transmitted to human beings via food items or household products contaminated through urine and faeces of the reservoir, Mastomys rats. Human transmission is Lassa fever is occurred by direct contact and estimated 80% of infected persons are asymptomatic [1, 2]. Besides the direct contact, laboratory transmission of Lassa fever is also a topical issue taking place in hospitals with inadequate sanitization and preventive measures [3]. In drastic cases, Lassa virus mainly affect foremost organs like liver, spleen and kidneys along with 1% fatality rate [1]. In case of severe infection profile, expiration, maternal death or loss of fetus (during the third trimester of pregnancy)
usually happen within 14 days of commencement of fatal symptoms. Lassa fever has the incubation period of 6–21 days approximately and for symptomatic patients, symptoms like fever, weakness, malaise, headaches, sore throat, vomiting,

nausea, diarrhea, swelling of face, abdominal pain, fluid retention in lungs, blood exposure in mouth, nose, sexual organs, GI tract, low blood pressure, shock etc are noticeable predominantly [1, 4, 5].

The first detection of Lassa fever infection was ensued in the year 1950, however the first confirmed case of Lassa was documented in the year 1969, in the town Lassa, situated in Borno state of Nigeria [6, 7]. According to the report of WHO, the largest outbreak of Lassa fever up to now was reported in Nigeria in the year 2018 [8]. According to Nigeria Centre for Disease Control and Prevention report, an estimated 5669 cases have suspected, along with 832 confirmed cases and 152 death cases from January 1, 2024 to April 14, 2024 [9]. Indeed, the number of incidences is higher originally than the reported cases, since in most areas of West Africa the surveillance of the transmission could not be performed [10, 11]. There is effective vaccine against Lassa fever until now; Ribavirin is supposed to be an antiviral medication against the infection if is prescribed early stages of infection [1]. WHO recommends some preventive measures for community like storing grain, foodstuffs in some rodent-proof containers, disposing of garbage regularly away from community, cleaning of households and keeping cats and some personal preventive measures like maintaining basic hand, face and respiratory hygiene, safe sex practice, safe injection practices, safe burial practices etc.

Mathematical modeling of an infected disease is beneficial to understand the kinetics of the overall disease pattern, course of infection, several factors and parameters affecting the dynamics of the disease, and possible control of the infection by means of several control strategies. Several mathematical models of Lassa fever have been developed [4, 8, 11 – 15] portraying the transmission stages and dynamical traits of infection considering either vector population, or human population or both populations. Some of these models employed different preventive strategies, in particular isolation, reducing intrahuman contact and contact with rodents, cleaning of environment etc in declining the infectiousness and to control the Lassa fever transmission. A very few models studied the impact of memory or another hereditary profile in contagion of Lassa fever [14, 15, 18]. Motivated by these works, in our present study we develop a deterministic, compartmental, integer-order mathematical model of Lassa fever transmission by upgrading the existing models of Lassa fever contagion proposed by [3, 16, 17]. Furthermore, we upgrade the proposed integer-order model to a Caputo fractional-order model incorporating the impact of immunological memory.

Present article is calibrated as follows: Section 2 is comprised of the mathematical modeling of Lassa fever transmission. In Section 3, the epidemic system is upgraded to its fractional-order counterpart in Caputo sense. In Section 4, local dynamics of the epidemic system around both the steady states is studied. Section 5 is designed with various numerical simulations. In Section 6, conclusions regarding comprehensive results are attached.

2. Lassa fever model

Taking into account both the vector (Mastomys rat) population and human population, we proposed an upgraded [3, 16, 17], non–linear deterministic, integer-order mathematical model comprising of the transmission dynamics of Lassa fever. We consider five compartments as (i) susceptible rats ($S_m(t)$), (ii) infected rats ($I_m(t)$), (iii) susceptible humans ($S_h(t)$), (iv) infected humans ($I_h(t)$) and (v) recovered humans ($R_h(t)$), at time t (days). Our proposed coupled system of nonlinear system of ODE equations is as follows:

$$\begin{aligned}
\frac{dS_m}{dt} &= \Lambda_m(1-\theta) - \beta_3 S_m I_m - \mu_m S_m, \\
\frac{dI_m}{dt} &= \Lambda_m\theta + \beta_3 S_m I_m - (\delta_m + \mu_m) I_m, \\
\frac{dS_h}{dt} &= \Lambda_h - \beta_1 S_h I_m - \beta_2 I_h S_h - \mu_h S_h, \\
\frac{dI_h}{dt} &= \beta_1 S_h I_m + \beta_2 I_h S_h - (\gamma_h + \delta_h + \mu_h) I_h, \\
\frac{dR_h}{dt} &= \gamma_h I_h - \mu_h R_h,
\end{aligned}$$

with epidemiologically feasible non-negative initial conditions:

$$S_m(0) = S_{m0} \geq 0, I_m(0) = I_{m0} \geq 0, S_h(0) = S_{h0} \geq 0, I_h(0) = I_{h0} \geq 0, R_{h0} = R_{h0} \geq 0.$$

Here we assume that all the model parameters are positive and their epidemiological descriptions are enlisted in Table 1. The time of infection is measured in days. The flow of Lassa fever transmission dynamics is depicted in the Figure 1.

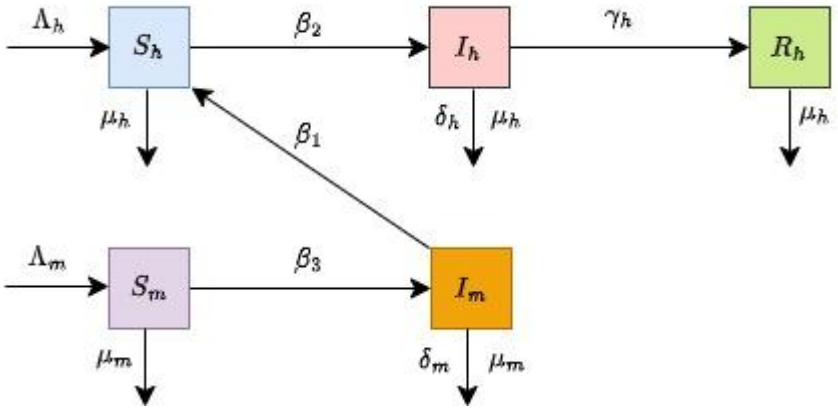

Figure 1. The figure is portraying the transmission dynamics of System (1).

Table 1: Model parameters and their epidemiological descriptions of the System (1)

Parameters	Definitions	Values
Λ_m	Constant recruitment of rats	500
Λ_h	Constant recruitment of susceptibles	2000
β_1	Rate of zoonotic transmission	0.0008
β_2	Rate of intrahuman transmission	0.0008
β_3	Rate of rat to rat infection	0.00714
μ_m	Rate of natural death of rats	0.00038
μ_h	Rate of natural death of humans	0.00005
δ_m	Rate of disease induced death of rats	0.0001
δ_h	Lassa fever induced death of humans	0.00074
γ_h	Rate of recovery of humans	0.84
θ	Fraction of new recruitment of infected rats	0.6

3. Fractional-order advancement of the system

Intending to upgrade our proposed integer-order model of Lassa fever [1] into a fractional-order differential equations (FODs) model, first we are recalling two widely used definitions of FODs, namely Riemann-Liouville and Caputo derivatives [19–21]. These two definitions are mostly used in analyzing real-life dynamical characteristics of a mathematical model like hereditary properties, genetic profile, memory etc. In our present work, we implement fractional-order derivatives in Caputo sense to convert our proposed integer-order model into a fractional one cogitating the huge advantages of Caputo fractional derivatives in solving real-life problems.

Definition 1. [22] *Let a function* $f \in C^n[0,p]$, *space of all* n *times continuously differentiable functions in* $[0,p]$. *Then the Caputo fractional-order derivative of* f *is defined by* ${}_0^C D_t^{\vartheta} f(t) = \frac{1}{\Gamma(n-\vartheta)} \int_0^t \frac{f^{(n)}(y)}{(t-y)^{(\vartheta-n+1)}} dy, n-1 < \vartheta \leq n \in N \text{ and } t > 0,$

where $\Gamma(.)$ *is the well-known Gamma function and* ϑ *is the order of FODs. In particular, for* $0 < \vartheta \leq 1$, *the above definition could be written as* ${}_0^C D_t^{\vartheta} f(t) = \frac{1}{\Gamma(1-\vartheta)} \int_0^t \frac{f^{(n)}(y)}{(t-y)^{\vartheta}} dy.$

Our proposed Caputo fractional-order deterministic model of Lassa fever considering the influence of immunological memory is constructed as follows:

$$\begin{aligned}
{}_0^C D_t^\alpha S_m(t) &= \Lambda_m^\alpha - \beta_3^\alpha S_m(t) I_m(t) - \mu_m^\alpha S_m(t), \\
{}_0^C D_t^\alpha I_m(t) &= \beta_3^\alpha S_m(t) I_m(t) - (\delta_m^\alpha + \mu_m^\alpha) I_m(t), \\
{}_0^C D_t^\alpha S_h(t) &= \Lambda_h^\alpha - \beta_1^\alpha S_h(t) I_m(t) - \beta_2^\alpha I_h(t) S_h(t) - \mu_h^\alpha S_h(t), \\
{}_0^C D_t^\alpha I_h(t) &= \beta_1^\alpha S_h(t) I_m(t) + \beta_2^\alpha I_h(t) S_h(t) - (\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha) I_h(t), \\
{}_0^C D_t^\alpha R_h(t) &= \gamma_h^\alpha I_h(t) - \mu_h^\alpha R_h(t),
\end{aligned} \tag{3}$$

with the same non-negative initial condition (2) and α is the order of fractional-order derivatives, defined as the index of immunological memory such that $0 < \alpha \leq 1$. The epidemiological descriptions of the baseline parameters and their values used for numerical simulation are enlisted in the Table 1. The parametric values are collected from [1, 3, 5, 16, 17].

4. Basic characteristics of the Caputo fractional-order system

In this section, we investigate the well-posedness of the Caputo-fractional-order system (3) in addition to the non-negative initial conditions (2). In this regard, we check the non-negativity of the solution trajectories of the Caputo fractional-order system (3) and their uniform boundedness.

4.1 Non-negativity of solutions

At first, we consider a set

$$\Phi = \left\{\mathcal{G}(t) \in \mathbb{R}_+^{\,5} : \mathcal{G}(t) \geq 0\right\} \text{ where } \mathcal{G}(t) = (S_m(t),\ I_m(t), S_h(t),\ I_h(t),\ R_h(t))$$

Next, to prove that all the solution trajectories of the Caputo fractional-order system (3) are non-negative and belong to the region Φ, we take help of the following theorem (established in the work of [23]).

Theorem 1. *Let us consider that the above defined function $f(t)$ and its Caputo fractional-order derivative ${}_0^C D_t^\alpha f(t)$ both belong to the metric space $C[p, q]$ and the condition $0 < \alpha \leq 1$ holds. Then, for all $t \in [p, q]$ the function $f(t)$ would be monotonically increasing if ${}_0^C f(t) \geq 0$ and the function $f(t)$ would be monotonically decreasing if ${}_0^C D_t^\alpha f(t) \leq 0$.*

Using the Theorem 1, we state the following corollary.

Lemma 1. *Suppose that $h(t) \in O[0, u]$ and ${}_0^C D_t^\alpha h(t) \in O$ for $0 < \alpha \leq 1$. If ${}_0^C D_t^\alpha h(t) \geq 0$ for all $t \in (0, u)$, then function h is non-decreasing and if ${}_0^C D_t^\alpha h(t) < 0$ for all $t \in (0, u)$, then the function h in non-increasing for all $t \in (0, u)$.*

Now, using the above Theorem 1 and Lemma 1, we prove the positivity of all the solutions of the Caputo fractional-order system (3).

Theorem 2. *All the solution trajectories of the Caputo fractional-order system* (3) *along with non-negative initial conditions* (2) *are positively oriented and belong to the region Φ (defined previously).*

Proof. From the system of Caputo fractional-order equations [3], it is observed that

$${}_0^C D_t^\alpha S_m \Big| S_m = 0 = \Lambda_m^\alpha \geq 0, {}_0^C D_t^\alpha I_m \Big|_{I_m = 0} = 0 \geq 0, {}_0^C D_t^\alpha S_{h_{S_h = 0}} = \Lambda_h^\alpha \geq 0,$$

$${}_0^C D_t^\alpha I_h \Big| I_h = 0 = \beta_1^\alpha S_h(t) I_m(t) \geq 0, {}_0^C D_t^\alpha R_h \Big|_{R_h = 0} = \gamma_h^\alpha I_h(t) \geq 0.$$

Thus, for any time $t \in (0, \infty)$, it is obtained that $S_m(t) \geq 0$, $I_m(t) = 0$, $S_h(t) \geq 0$, $I_h(t) \geq 0$, and $R(t) \geq 0$. Hence, all the solutions $(S_m, I_m, S_h, I_h, R_H)$ are positively oriented in R_{+^5}. □

4.2 Uniform boundedness of solutions

Theorem 3. *Every solution of the Caputo fractional-order system* (3) *with non-negative initial conditions* (2) *initiating in the region* Φ *is uniformly bounded in* R_{+^5}.

Proof. Firstly, by summing up the first two equations of the Caputo fractional-order system (3), it is obtained that

$$\begin{aligned} {}_0^C D_t^\alpha N_m(t) &= \Lambda_m^\alpha - \vartheta_m^\alpha\big(S_m(t) + I_m(t)\big) \\ i.e. \qquad &\leq \Lambda_m^\alpha - \vartheta_m^\alpha N_m(t), \end{aligned}$$

where $\vartheta_m^\alpha = max\{\mu_m^\alpha, \mu_m^\alpha + \delta_m^\alpha\}$, since ${}_0^C D_t^\alpha N_m(t) + \vartheta_m N_m(t) \leq \Lambda_m^\alpha$ and ${}_0^C D_t^\alpha N_h(t) + \vartheta_h N_h(t) \leq \Lambda_h^\alpha$. Now, it could be obtained

$$N_m(t) \leq (N(0) - \Lambda_m^\alpha) E_\nu(-\vartheta_m^\alpha t^\alpha) + \frac{\Lambda_m^\alpha}{\vartheta_m^\alpha}, \forall t \in .$$

Accordingly, $N_m(t) \to \Lambda_m^\alpha \vartheta^\alpha$ as $t \to \infty$. Now, adding the last three equations of the Caputo fractional-order system (3), we have

$$\begin{aligned} {}_0^C D_t^\alpha N_h(t) &= \Lambda_h^\alpha - \vartheta_h^\alpha\big(S_h(t) + I_h(t) + R(t)\big) \\ i.\ e. \qquad &\leq \Lambda_h^\alpha - \vartheta_h^\alpha N_h(t) \end{aligned}$$

where $\vartheta_h^\alpha = max\{\mu_h^\alpha, (\mu_h^\alpha + \mu_h^\alpha)\}$ since, ${}_0^C D_t^\alpha N_h(t) + \vartheta_h N_h(t) \leq \Lambda_h^\alpha$and $CD_t^\alpha N_h(t) + \vartheta_h N_h(t) \leq \Lambda_h^\alpha$. Thus, it could be obtained that

$$N_h(t) \leq (N(0) - \Lambda_h^\alpha) E_\nu(-\vartheta_h^\alpha t^\alpha) + \frac{\Lambda_h^\alpha}{\vartheta_h^\alpha}, \forall t \in .$$

Accordingly, $N_h(t) \to \Lambda_h^\alpha \vartheta^\alpha$ as $t \to \infty$. Consequently, all the solutions $(S_m, I_m, S_h, I_h, R_H)$ of the Caputo fractional-order system are uniformly bounded in the region:

$$\Theta = \left\{ H \in R^5 : S_m(t) + I_m(t) \leq \frac{\Lambda_m^\alpha}{\vartheta_m^\alpha} \wedge S_h(t) + I_h(t) + R_h \leq \frac{\Lambda_h^\alpha}{\vartheta_h^\alpha} \right\}.$$

The region Θ is positively invariant and attracting and the well-posedness of the Caputo fractional-order system is proved in this manner.

5 Steady States and basic reproduction number of the model

In this Section, we investigate the feasible equilibrium points executed by the Caputo fractional-order Lassa fever system (3) and their existence criteria. It is seen that the system possesses two feasible steady states -

(i) An infection-free equilibrium point - $A_0 = \left(\frac{\Lambda_m}{\mu_m}, 0, \frac{\Lambda_h}{\mu_h}, 0, 0\right)$, which exists irrespective of any epidemiological condition.
(ii) Endemic equilibrium point - $A^* = (S_m, I_m, S_h, I_h, R_h)$, whose existence condition would be studied.

5.1 Basic reproduction number of the system

With the aim of finding the basic reproduction number of the Caputo fractional-order system (3), we take help of the next-generation matrix method [24]. Basic reproduction number, a threshold, is essential to estimate the conditions for spreading of an infection, and to predict the future course of outbreak. In this aspect, we construct two matrices F and V representing the flow of new Lassa fever infection and the transition of infection between infected compartments at the infection-free steady state A_0 as follows:

$$F = \begin{pmatrix} \frac{\Lambda_m^\alpha \beta_3^\alpha}{\mu_m} & 0 \\ \frac{\Lambda_h^\alpha \beta_1^\alpha}{\mu_h} & \frac{\Lambda_h^\alpha \beta_2^\alpha}{\mu_h} \end{pmatrix}, and\ V = \begin{pmatrix} \delta_m^\alpha + \mu_m^\alpha & 0 \\ 0 & \gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha \end{pmatrix}.$$

The basic reproduction number, R_0 (say), is the spectral radius of the next-generation matrix FV^{-1} and is computed as

$$R_0 = max\{R_m, R_h\}.$$

We may express the basic reproduction number, R_0 as the combination of another two thresholds - R_m representing the control reproduction number for rat population and R_h representing the control reproduction number for human population respectively given as below:

$$\begin{aligned} R_m &= \frac{\beta_3^\alpha \Lambda_m^\alpha}{\mu_m^\alpha(\delta_m^\alpha + \mu_m^\alpha)}, \\ R_h &= \frac{\beta_2^\alpha \Lambda_h^\alpha}{\mu_h^\alpha(\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha)}. \end{aligned}$$

5.2 Existence of endemic steady state

The components of the endemic steady state are given as follows:

$$\begin{aligned} S_m &= \frac{\delta_m^\alpha + \mu_m^\alpha}{\beta_3^\alpha}, \\ I_m &= \frac{\mu_m^\alpha}{\beta_3^\alpha}(R_m - 1), \\ S_h &= \frac{\Lambda_h^\alpha \beta_3^\alpha(\delta_m^\alpha + \mu_m^\alpha)}{\beta_1^\alpha \mu_m^\alpha(\delta_m^\alpha + \mu_m^\alpha)(R_m - 1) + \beta_2^\alpha I_h + \mu_h^\alpha}, \\ R_h &= \frac{\gamma_h^\alpha I_h}{\mu_h^\alpha} \end{aligned}$$

and I_h satisfies the function $F(I_h) = 0$, where the function $F(I_h)$ is defined as

$$F(I_h) = F(I_h) = (\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha)\beta_2^\alpha I_h^2 + \left(\frac{\beta_1^\alpha \mu_m^\alpha(\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha)(R_m - 1)}{\beta_3^\alpha} + (\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha)\mu_h^\alpha - \Lambda_h^\alpha \beta_2^\alpha\right) I_h - \frac{\Lambda_h^\alpha \beta_1^\alpha \mu_m^\alpha(R_m - 1)}{\beta_3^\alpha}.$$

Accordingly, it is noticeable that $F(0) < 0$ while $R_m > 1$. Moreover, $F(I_h) \to +\infty$ as $I_h \to +\infty$. Hence, the endemic equilibrium would exist whenever $R_m > 1$.

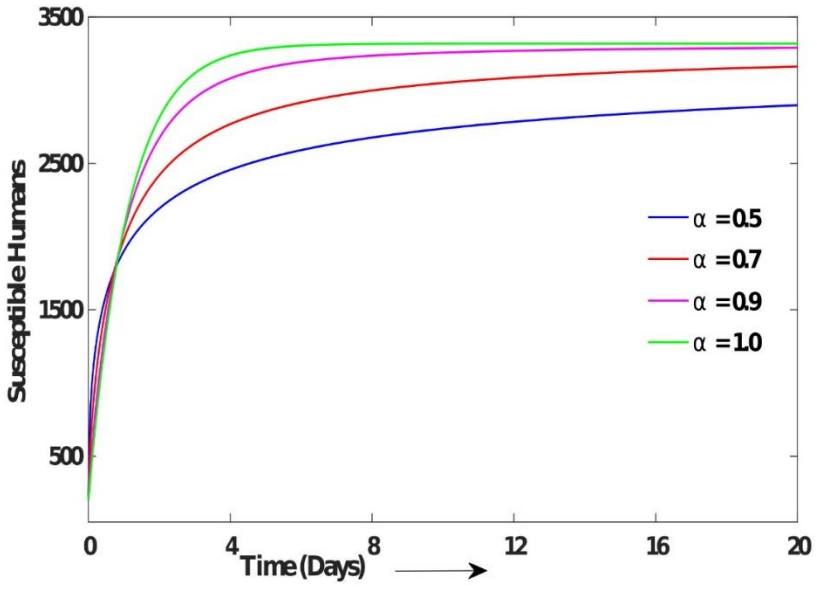

Figure 2. The figure is showing the time series evolution in the infected human population of the Caputo fractional-order system (3) for different values of immunological memory, α = 0.5, 0.7, 0.9, 1.0 and taking other baseline parameter values same as in Table 1.

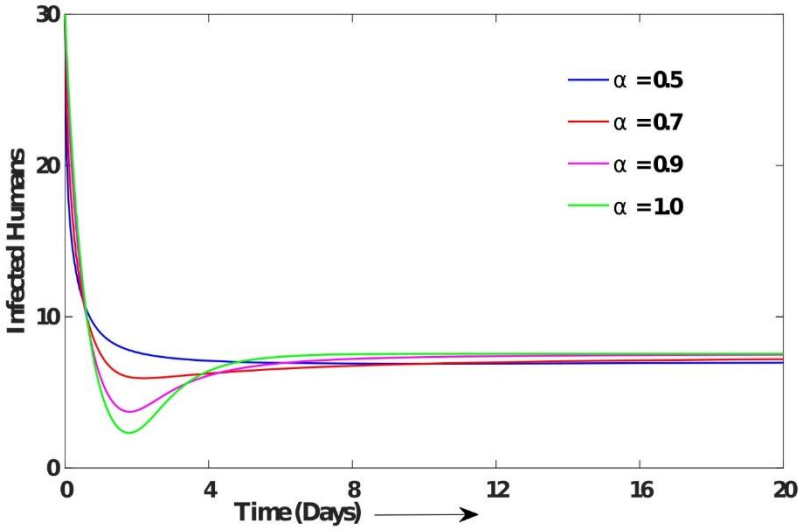

Figure 3: The figure is showing the time series evolution in the infected human population of the Caputo fractional-order system (3) for different values of immunological memory, α = 0.5, 0.7, 0.9, 1.0 and taking other baseline parameter values same as in Table 1.

6 Local dynamics of the system

In this section, we would study the local asymptotic stability of the Caputo fractional-order system [3] around the infection-free equilibrium point $A_0 = \left(\frac{\Lambda_m}{\mu_m}, 0, \frac{\Lambda_h}{\mu_h}, 0, 0\right)$ and the endemic equilibrium point $A^* = (S_m, I_m, S_h, I_h, R_h)$.

Theorem 4. *The Caputo fractional-order system* [3] *of Lassa fever would be locally asymptotically stable around the infection-free equilibrium point* $A_0 = \left(\frac{\Lambda_m}{\mu_m}, 0, \frac{\Lambda_h}{\mu_h}, 0, 0\right)$ *while* $R_0 < 1$*; otherwise, instability occurs in the system* [3].

Proof. In order to determine the local asymptotic stability of the Caputo fractional-order system [3] around the infection-free equilibrium point $A_0 = \left(\frac{\Lambda_m}{\mu_m}, 0, \frac{\Lambda_h}{\mu_h}, 0, 0\right)$, first we have to compute the Jacobian matrix of the system [3] at the infection-free equilibrium point A_0 as

$$J_{A_0} = \begin{pmatrix} -\mu_m & a_{12} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & a_{32} & -\mu_h & a_{34} & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 \\ 0 & 0 & 0 & \gamma_h & -\mu_h \end{pmatrix},$$

where the components $a_{12}, a_{22}, a_{32}, a_{42}, a_{34}$ and a_{44} are computed as

$$\begin{aligned} a_{12} &= -\frac{\beta_3^\alpha \Lambda_m^\alpha}{\mu_m^\alpha}, a_{22} = \frac{\beta_3^\alpha \Lambda_m^\alpha}{\mu_m^\alpha} - (\delta_m^\alpha + \mu_m^\alpha), \\ a_{32} &= -\frac{\beta_1^\alpha \Lambda_h^\alpha}{\mu_h^\alpha}, a_{34} = -\frac{\beta_2^\alpha \Lambda_h}{\mu_h}, \\ a_{42} &= \frac{\beta_1 \Lambda_h}{\mu_h}, a_{44} = \frac{\beta_2 \Lambda_h}{\mu_h} - (\gamma_h + \delta_h^\alpha + \mu_h^\alpha). \end{aligned}$$

It is noticeable that the Jacobian matrix J_{A_0} possesses three purely real and strictly negative eigenvalues namely $-\mu_m^\alpha, -\mu_h^\alpha, -\mu_h^\alpha$. Furthermore, the rest two eigenvalues are $(\delta_m^\alpha + \mu_m^\alpha)(R_m - 1)$ and $(\gamma_h^\alpha + \delta_h^\alpha + \mu_h^\alpha)(R_h - 1)$. Thus, these two eigenvalues would be real and negative or having negative real parts if and only if $R_m < 1$ and $R_h < 1$. Taking help of the Theorem 5 proposed in the work of Samui et al. [21], it is seen that $|arg(\lambda_i)| = \pi > \frac{\pi\alpha}{2}, i = 1,2,3,4,5$ and $0 < \alpha \leq 1$. Consequently, the Caputo fractional-order system [3] is locally asymptotically stable around the infection-free equilibrium point on condition that $R_m < 1$ and $R_h < 1$. □

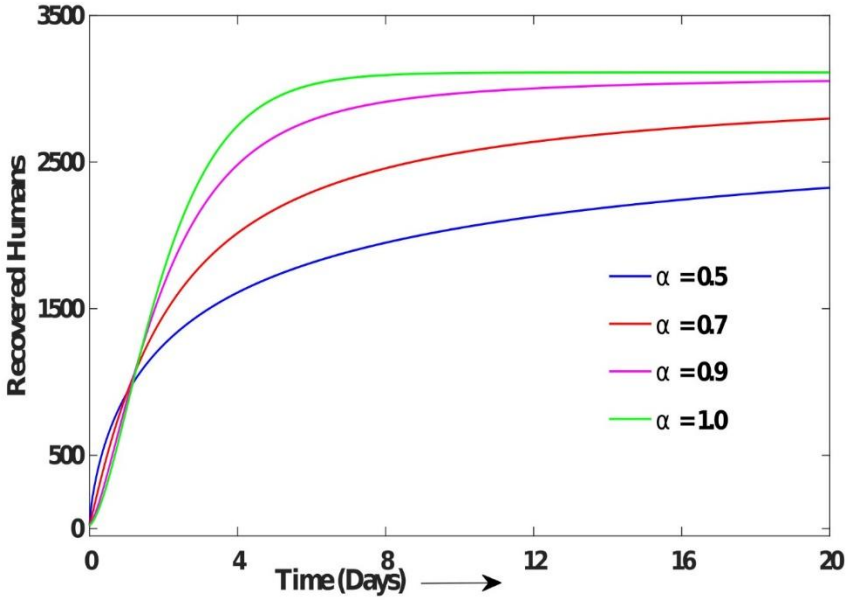

Figure 4: The figure is showing the time series evolution in the recovered human population of the Caputo fractional-order system (3) for different values of immunological memory, α = 0.5, 0.7, 0.9, 1.0 and taking other baseline parameter values same as in Table 1.

Theorem 5. *The Caputo fractional-order system* [3] *of Lassa fever would be locally asymptotically stable around the endemic equilibrium point* $A^* = (S_m, I_m, S_h, I_h, R_h)$ *while* $R_0 > 1$*; otherwise, instability occurs in the system* [3].

Proof. To determine the local asymptotic stability of the system [3] around the the endemic equilibrium point $A^* = (S_m, I_m, S_h, I_h, R_h)$, first we compute the Jacobian matrix of the system [3] about the endemic equilibrium point (EE) as

$$J_{A^*} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & a_{42} & a_{43} & a_{44} & 0 \\ 0 & 0 & 0 & \gamma_h & -\mu_h \end{pmatrix},$$

where we consider

$$\begin{aligned} a_{11} &= -\mu_m - \beta_3 I_m, a_{12} = \beta_3 S_m, a_{21} = -\beta I_m \\ a_{22} &= -\beta_3 S_m - (\delta_m + \mu_m), a_{32} = -\beta_1 S_h, \\ a_{33} &= -\mu_h - \beta_1 I_m - \beta_2 I_h, a_{34} = -\beta_2 S_h, a_{42} = \beta_1 S_h, \\ a_{32} &= \beta_2 S_h - (\gamma_h + \delta_h + \mu_h), a_{42} = -\beta_1 I_m + \beta_2 I_h. \end{aligned}$$

It is seen that the Jacobian matrix J_{A^*} have one purely real and strictly negative eigenvalues $-\mu_h$. Further, the rest four eigenvalues can be obtained from the following characteristic equation

$$\lambda^4 + \zeta_1 \lambda^3 + \zeta_2 \lambda^2 + \zeta_3 \lambda + \zeta_4 = 0, \tag{4}$$

where

$$\begin{aligned} \zeta_1 &= -a_{11} - a_{22} - a_{33} - a_{44}, \\ \zeta_2 &= a_{11}a_{22} + a_{11}a_{33} + a_{11}a_{44} + a_{22}a_{33} + a_{22}a_{44} + a_{33}a_{44} - a_{12}a_{21} - a_{34}a_{43}, \\ \zeta_3 &= a_{12}a_{21}a_{33} + a_{12}a_{21}a_{44} + a_{11}a_{34}a_{43} + a_{22}a_{34}a_{43} - a_{11}a_{22}a_{44} - a_{11}a_{22}a_{33} - a_{22}a_{33}a_{44} - a_{11}a_{33}a_{44}' \\ \zeta_4 &= a_{11}a_{22}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43} - a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43} \end{aligned}$$

Next, we present two propositions to study the local asymptotic stability of the system [3] around the endemic equilibrium.

Proposition 1. *If the four eigenvalues* $\lambda_i, i = 1,2,3,4$ *of the Jacobian matrix* J_E *satisfies the condition* $|arg(\lambda)| > \frac{\lambda\pi}{2}$ *(apart from the eigenvalue* $-\mu_h$

which is already negative), then the epidemic system [3] *is locally asymptotically stable around the EE.*

Next, we determine the discriminant of the characteristic equation (4) as:

$$\Upsilon(\varphi) = \begin{vmatrix} 1 & \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & 0 & 0 \\ 0 & 1 & \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & 0 \\ 0 & 0 & 1 & \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 \\ 4 & 3\zeta_1 & 2\zeta_2 & \zeta_3 & 0 & 0 & 0 \\ 0 & 4 & 3\zeta_1 & 2\zeta_2 & \zeta_3 & 0 & 0 \\ 0 & 0 & 4 & 3\zeta_1 & 2\zeta_2 & \zeta_3 & 0 \\ 0 & 0 & 0 & 4 & 3\zeta_1 & 2\zeta_2 & \zeta_3 \end{vmatrix},$$

$$= \zeta_1^2\zeta_2^2\zeta_3^2 - 4\zeta_1^2\zeta_2^3\zeta_4 - 4\zeta_1^3\zeta_3^2 + 18\zeta_1^3\zeta_2\zeta_3\zeta_4 - 27\zeta_1^4\zeta_2^2 + 4\zeta_2^3\zeta_3^2 + 16\zeta_2^4\zeta_4 + 18\zeta_1\zeta_2\zeta_3^3 - 80\zeta_1\zeta_2^2\zeta_3\zeta_4 - 6\zeta_1^2\zeta_2^2\zeta_4 + 144\zeta_1^2\zeta_2\zeta_4^2 - 27\zeta_3^4 + 144\zeta_2\zeta_3^2\zeta_4 - 128\zeta_2^2\zeta_4^2 - 192\zeta_1\zeta_3\zeta_4^2 + 256\zeta_4^3$$

In terms of the discriminant $\Upsilon(\varphi)$, we construct the following proposition to study the local asymptotic stability of the system [3] around the endemic equilibrium point.

Proposition 2. (a) *The epidemic system* [3] *is locally asymptotically stable around the EE if* $\Upsilon(\varphi) > 0$, *in addition to the conditions* (i) $\zeta_1 > 0$, (ii) $\zeta_1\zeta_2 > \zeta_3$, *and* (iii) $\zeta_1\zeta_2\zeta_3 - \zeta_1^2\zeta_4 - \zeta_3 > 0$.

(b) *The epidemic system* [3] *is locally asymptotically stable around the EE for* $\alpha \in (0.5, 1)$, *if* $\Upsilon(\varphi) < 0$, *in addition to the conditions* (i) $\zeta_1 > 0$, (ii) $\zeta_2 > 0$, (iii) $\zeta_1\zeta_2 > \zeta_3$, *and* (iv) $\zeta_1\zeta_2\zeta_3 - \zeta_1^2\zeta_4 - \zeta_3 = 0$.

(c) *The epidemic system* [3] *is unstable around the EE for* $\alpha > 2/3$, *if* $\Upsilon(\varphi) < 0$ *together with the conditions* (i) $\zeta_1 < 0$, (ii) $\zeta_2 < 0$, *and* (iii) $\zeta_3 < 0$.

□

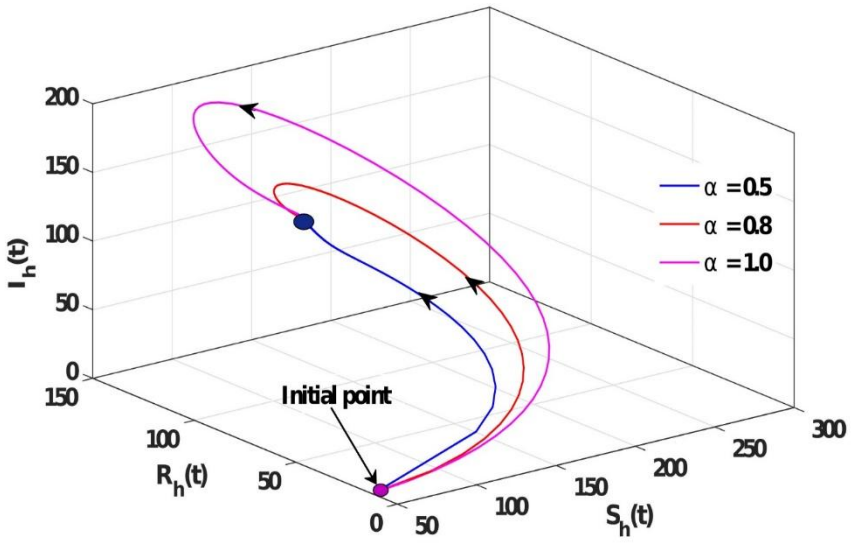

Figure 5: The figure is showing the time series evolution in the recovered human population of the Caputo fractional-order system (3) for different values of immunological memory, α = 0.5, 0.7, 0.9, 1.0 and taking other baseline parameter values same as in Table 1.

7 Numerical simulation

In this section, we numerically validate our proposed Caputo fractional-order Lassa fever model (3) using the MATLAB software with baseline parameter values enlisted in the Table 1. Intended to capture the dynamics of the Caputo fractional-order model, we vary the value of the immunological memory, α. Indeed, for $\alpha = 1$, the Caputo fractional-order system would shrink to its integer-order counterpart. In Figure 2, Figure 3 and Figure 4, the behaviors of the Caputo fractional-order system for different values of the

immunological memory, $\alpha = 0.5,0.7,0.9,1.0$ are observed in case of three populations - susceptible, infected and recovered human population respectively. It is observed that for higher value of immunological memory, the load of Lassa fever infection would be declined. In Figure 5, the global asymptotic stability of the Caputo fractional-order system is depicted in the phase space $S_h - R_h - I_h$ showing the gradual declination of the Lassa fever infection for increasing values of immunological memory.

8 Conclusions

In epidemiology, Caputo fractional-order differential equations are referring to be the most interesting tool in analyzing dynamics of an infectious disease, disease pattern, course of infection and future course of outbreak. In our present research study, we formulate a deterministic, five compartmental model of Lassa fever taking into account both the human population and rat population. Furthermore, we perturb the integer-order Lassa model into a Caputo fractional-order model accounting the impact of immunological memory in disease progression, mitigation and curtailing of Lassa fever infection.

Data accessibility. All data generated or analyzed during this study are included in this article.

Authors' contributions.

Avik Bag: Formal analysis, Conceptualization, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization.

Chandan Maji: Formal analysis, Conceptualization, Software, Validation, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision.

Conflict of interest declaration. The author declares that there is no conflict of interests regarding the publication of this article.

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